Universidad Nacional Autónoma de México
Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas

Operator Theory Analysis and Mathematical Physics 2020

Mexico City 8–14 January 2020

Book of Abstracts

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Operator Theory, Analysis and Mathematical Physics

Mexico City 8–14 January 2020

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Preface

The OTAMP conferences have been organized every second year from 2002 to 2016 in various European cities: Bedlewo (2002, 2004, 2008, 2010), Lund (2006), Barcelona (2012), Stockholm (2014), Saint Petersburg (2016).

The main aim of the conference is to bring together leading experts and young promising researchers from all over the world working on mathematical problems within the framework of mathematical physics. The idea is to facilitate exchange of ideas, and develop future and already existing collaborations.

This edition of the conference will honor Ricardo Weder, his influential work on mathematical physics, and his crucial role in the foundation and development of the mathematical physics community in Mexico.

Ricardo Weder obtained his undergraduate degree at Universidad Nacional de Rosario, Argentina, in 1970, and his PhD at the K U Leuven, Belgium, in 1974. After postdoctoral stays at the Universities of Harvard and Princeton, he joined in 1978, the Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas and the Facultad de Ciencias, both of Universidad Nacional Autónoma de México (UNAM). He has been a visiting professor at numerous universities and research institutes in Argentina, Europe, Japan, and the United States. He has offered plenary lectures at many international conferences. He was awarded the Premio Alejandrina of research and the Silver Medal of Sociedad Matemática Mexicana, among other distinctions. He organized several international conferences , schools, and workshops. He is a member of several professional societies, in particular of International Association of Mathematical Physics and of Sociedad Matemática Mexicana. He initiated in Mexico the research in mathematical physics and he played a crucial role in the foundation and on the development of the mathematical physics community in México. His area of research is functional analysis and mathematical physics. In particular, direct and inverse spectral and scattering theory, quantum mechanics and classical wave

propagation. He has taught courses in analysis and on mathematical physics to many generations of students at UNAM. He has been the thesis advisor of many students, at the undergraduate and graduate levels.

TALKS

CAUCHY – DE BRANGES SPACES AND RANK ONE PERTURBATIONS OF COMPACT NORMAL OPERATORS

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Abstract

I will discuss how the classical de Branges theory of Hilbert spaces of entire functions can be extended to a more general setting of Cauchy – de Branges spaces. In particular, new versions of de Branges Ordering Theorem for nearly invariant subspaces in Cauchy – de Branges spaces will be reviewed. Also, I plan to talk about how the obtained results can be applied to the study of rank one perturbations of compact normal operators via a functional model.
This is joint work with A. Baranov and Yu. Belov.

THE INVERSE SCATTERING FOR THE MATRIX SCHRÖDINGER OPERATOR ON THE HALF LINE

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Abstract

The matrix Schrödinger equation is considered on the half line with the general selfadjoint boundary condition. It is assumed that the matrix potential is integrable, selfadjoint, and has a finite first moment. The corresponding scattering data set is constructed, and such scattering data sets are characterized by providing a set of necessary and sufficient conditions assuring the existence and uniqueness of the one-to-one correspondence between the scattering data set and the input data set containing

the potential and the boundary condition. The work presented here provides a generalization of the classical result by Agranovich and Marchenko from the Dirichlet boundary condition to the general selfadjoint boundary condition. The research is based on the joint work with Ricardo Weder of National Autonomous University of Mexico.

HIGH-VELOCITY ESTIMATES FOR THE SCATTERING OPERATOR AND AHARONOV-BOHM EFFECT IN THREE DIMENSIONS

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Abstract

We obtain high-velocity estimates with error bounds for the scattering operator of the Schrödinger equation in three dimensions with electromagnetic potentials in the exterior of bounded obstacles that are handlebodies. A particular case is a finite number of tori. We prove our results with time-dependent methods. We consider high-velocity estimates where the direction of the velocity of the incoming electrons is kept fixed as its absolute value goes to infinity. In the case of one torus our results give a rigorous proof that quantum mechanics predicts the interference patterns observed in the fundamental experiments of Tonomura et al. that gave conclusive evidence of the existence of the Aharonov-Bohm effect using a toroidal magnet. We give a method for the reconstruction of the flux of the magnetic field over a cross-section of the torus modulo 2π. Equivalently, we determine modulo 2π the difference in phase for two electrons that travel to infinity, when one goes inside the hole and the other outside it. For this purpose we only need the high-velocity limit of the scattering operator for one direction of the velocity of the incoming electrons. When there are several tori-or more generally handlebodies-the information that we obtain in the fluxes, and on the difference

of phases, depends on the relative position of the tori and on the direction of the velocities when we take the high-velocity limit of the incoming electrons. For some locations of the tori we can determine all the fluxes modulo 2π by taking the high-velocity limit in only one direction. We also give a method for the unique reconstruction of the electric potential and the magnetic field outside the handlebodies from the high-velocity limit of the scattering operator.
The talk is based on a joint work with R. Weder.

ON THE RESONANCE STATES OF ONE-DIMENSIONAL SCHRÖDINGER OPERATORS WITH FINITE POINT INTERACTIONS

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Abstract

In this talk we consider the formal one-dimensional Schrödinger operator

$$S_q u(x) = \left(-\frac{d^2}{dx^2} + q(x)\right) u(x), \quad x \in \mathbb{R}, \tag{1}$$

with a potential $q = q_0 + q_s$ consisting of a regular part $q_0 \in L^\infty(\mathbb{R})$ and a singular part

$$q_s(x) = \sum_{k=1}^{N} \left(\alpha_k \delta(x - x_k) + \beta_k \delta'(x - x_k)\right), \quad \alpha_k, \beta_k \in \mathbb{R},$$

with support on a finite set of points $\mathcal{X} = \{x_k\}_{k=1}^N$. We construct an extension $\mathcal{H}$ of operator (1) given in terms of only the regular potential q_0 and certain matrix conditions at every point of $\mathcal{X}$. We analyse the resonance states of the Schrödinger equation $\mathcal{H}\psi = \epsilon\psi$ for quite arbitrary regular potentias q_0, and derive a

characteristic equation $\varkappa(\epsilon) = 0$ that defines the complex energies $\epsilon_n = E_n - \mathrm{i}\Gamma_n/2$, $\Gamma_n \geq 0$, associated with resonances. We show that the characteristic function $\varkappa$ is an analytic function of the complex energy ϵ and determine its Taylor series.

EXISTENCE AND NON-EXISTENCE OF MINIMIZERS FOR POINCARÉ-SOBOLEV INEQUALITIES

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Abstract

In this talk I discuss the existence and non-existence of minimizers for a type of (critical) Poincaré-Sobolev inequalities. We show that minimizers do exist for smooth domains in R^d, and also for some polyhedral domains. On the other hand, we show the non-existence of minimizers in the rectangular isosceles triangle in R^2. This is joint work with Cristobal Vallejos (PUC, Chile) and Hanne Van Den Bosch (U. de Chile)

THE ESTIMATES $L_1 - L_\infty$ FOR THE REDUCED RADIAL EQUATION OF SCHRÖDINGER

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Abstract

Estimates of the type $L_1 - L_\infty$ for the Schrödinger equation on the line and on half-line with a regular potential V express the dispersive nature of the Schrödinger equation and are the essential elements in the study of initial value problems, the asymptotic

times for large solutions and scattering theory for the non linear Schrödinger equation in general and for other equations of nonlinear evolution. The estimates $L_p - L_{p'}$ express the dispersive nature of this equation.

We provide a transformation operator W that transforms the Reduced Radial Schrödinger Equation (RRSE) (whose main characteristic is the addition a singular term of quadratic order to a regular potential) into the Schrödinger Equation on Half-Line (RSEHL). Thus, W eliminates the singular term of quadratic order of potential in the asymptotic development towards zero and adds to the potential a bounded term and a term exponentially decrease fast enough in the asymptotic development towards infinity, which continues guaranteeing the uniqueness of the potential in the condition of the infinity boundary. The $L_1 - L_\infty$ for the (RRSE) are preserved under the transformation operator as in the case of (RSEHL).

EIGENVALUE ASYMPTOTICS FOR THE QUANTUM RABI MODEL

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Abstract

The quantum Rabi model is a quantum optics model describing the interac- tion of a two-level atom with a single quantized mode of an optical cavity's electromagnetic field. It depends on three parameters. It is also called Jaynes–Cummings model without the rotating-wave approximation. I will first explain how its spectral analysis reduces to that of some particular Jacobi matrices whose spectrum is discrete. Then I will present a three-term asymptotics expansion for the nth eigen- value of such a Jacobi matrix. This asymptotics transfers to the quantum Rabi model and allows to show that all the parameters of the model can be recovered from its spectrum.
This is joint work with Lech Zielinski.

INVERSE PROBLEMS FOR FRACTIONAL PARTIAL DIFFERENTIAL EQUATIONS. AN APPLICATION FOR THE PETROLEUM INDUSTRY

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Abstract

At 2010 there was a gigantic oil spill in the Gulf of Mexico caused by an accident on a British Petroleum (BP) deep-sea exploration rig. The worst environmental disaster in America. Then all the petroleum companies revised the mathematical models. PEMEX asked us for a model to obtain the pressure deficit for future exploration with fractional partial diferenctial equation. We will present the solution.

ESSENTIAL SPECTRUM FOR MAXWELL'S EQUATIONS

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Abstract

We study the essential spectrum of operator pencils associated with anisotropic Maxwell equations, with permittivity ϵ, permeability μ and conductivity σ, on finitely connected unbounded domains. The main result is that the essential spectrum of the Maxwell pencil is the union of two sets: namely, the spectrum of the pencil $\div((\omega\epsilon + i\sigma)\nabla\,\cdot\,)$, and the essential spectrum of the Maxwell pencil with constant coefficients.

A REGULARITY RESULT INFLUENCED BY GEOMETRY

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Abstract

In this talk we will present a regularity result for local minimizers of integral functionals. Motivated by Taheri's work (2003), we will establish bounds for the energy of local minimizers for which the geometry of the domain plays an important role. Furthermore, these bounds allow us to obtain full regularity up to the boundary for local minimizers under certain smallness conditions on the boundary datum.

ON ABSTRACT DIFFERENTIAL EQUATIONS OF ELLIPTIC TYPE WITH NONLOCAL BOUNDARY COEFFICIENT-OPERATOR CONDITIONS IN THE FRAMEWORK OF HÖLDERIAN SPACES: NONCOMMUTATIVE CASES

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Abstract

This work is devoted to the abstract study of operational second order differential equations of elliptic type with nonregular coefficient-operator boundary conditions in a non commutative framework. The study is performed when the second member f belong to $C^{\theta}([0,1]\,;X)$, with general $0<\theta<1$, X being a Banach space. We give some new results by using semigroups and

interpolation theory. Necessary and sufficient conditions of compatibility are established to obtain the classical solution. Maximal regularity properties are also studied.

Let X be a complex Banach space. We consider the following operational second order differential equation of elliptic type

$$u''(x) + Au(x) = f(x), \ x \in [0,1], \tag{2}$$

together with the following nonlocal coefficient-operator boundary conditions

$$\begin{cases} u(0) = u_0 \\ u(1) + Hu'(0) = u_{1,0}, \end{cases} \tag{3}$$

where A is a closed linear operator with domain $D(A)$ not necessarily dense in X and H is a closed linear operator with domain $D(H)$, $f \in C^{\theta}([0,1]; X)$ with $0 < \theta < 1$ and $u_0, u_{1,0}$ are given elements of X. As we will see, equation (2) is the abstract writing of $\Delta u = f$ for instance.

Our main goal is to seek a classical solution of the problem (2)-(3), that is a function u such that

$$\begin{cases} i)\ u \in C^2([0,1]; X) \cap C([0,1]; D(A)) \\ ii)\ u(0) \in D(H) \\ iii)\ u \text{ satisfies (2)-(3).} \end{cases}$$

We can also seek for a classical solution u satisfying the maximal regularity property $u'', Au \in C^{\theta}([0,1]; X)$.

PERIODIC PDES WITH CRITICAL CONTRAST: UNIFIED APPROACH TO HOMOGENISATION AND LINKS TO TIME-DISPERSIVE MEDIA

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Abstract

I shall discuss a novel approach to the homogenisation of high-contrast periodic PDEs, which yields an explicitly construction

of their norm-resolvent asymptotics. A practically relevant outcome of this result is that it interprets composite media with micro-resonators as a class of time-dispersive media.
This is joint work with Yulia Ershova and Alexander Kiselev.

INVERSE PROBLEM WITH TRANSMISSION EIGENVALUES FOR THE JACOBI SYSTEM

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Abstract

The Jacobi system with the Dirichlet boundary condition is considered on a half-line lattice when the coefficients are real valued. The inverse problem of recovery of the coefficients from various data sets containing the so-called transmission eigenvalues is analyzed. The Marchenko method and the Gel'fand-Levitan method are utilized to solve the corresponding inverse problem.
This is a joint work with Tuncay Aktosun and Vassilis G. Papanicolaou.

PHASE SEPARATION, OPTIMAL PARTITIONS AND NODAL SOLUTIONS TO THE YAMABE EQUATION ON THE SPHERE

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Abstract

We study a weakly coupled nonlinear elliptic system that arises as a model in many physical phenomena, for example, in the

study of standing waves for a mixture of Bose-Einstein condensates of M-hyperfine states which overlap in space. In the *competitive* case (where the interaction of the particles within each single state is attractive and the interaction between particles in any two different states is repulsive) the condensates separate spatially, giving rise to a partition into M disjoint domains.

Apart from its physical relevance, this phenomenon has interesting geometrical applications. The one we present in this talk concerns a fundamental question in conformal geometry, known as the *Yamabe problem*. Through the analysis of the limit profiles of the components of solutions to suitable competitive elliptic systems, we obtain the existence of an optimal M-partition for the Yamabe equation on the round sphere. We show that there is a one-to-one correspondence between such partitions and sign-changing solutions to the Yamabe equation on the sphere with precisely M nodal domains, having certain symmetry properties.

This is an example of how ideas coming from physics can be applied to tackle mathematical problems.
These results are joint work with Alberto Saldaña (IM-UNAM) and Andrzej Szulkin (Stockholm University).

QUANTUM GRAPH MODELS OF BILAYER AND TRILAYER GRAPHENE

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Abstract

A quantum graph model is extended to include bilayer and trilayer graphene. We rigorously derive some basic properties observed by physicists, e.g., the presence or absence of Dirac cones in their valence and conducting bands.

ON THE EXACT CONTROLLABILITY OF COUPLED ONE DIMENSIONAL WAVE EQUATIONS

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Abstract

The focus of this conference is the exact controllability of a system of N one-dimensional coupled wave equations when the control is exerted on a part of the boundary by means of one control. We give a Kalman condition (necessary and sufficient) and give a description of the attainable set. In general, this set is not optimal, but can be refined under certain conditions.

ON SOME FAMILIES OF EXACTLY SOLVABLE SCHRÖDINGER OPERATORS

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Abstract

I will discuss various realizations of one-dimensional Schrödinger operators with $1/x^2$ and $1/x$ potentials as closed operators on $L^2[0,\infty[$. It is natural to organize them into holomorphic families, allowing for complex coupling constants. Their properties are sometimes quite surprising: they undergo "phase transitions", they show various patterns of the action of the "renormalization group", singularities appear unexpectedly.

BISPECTRALITY OF LAGUERRE AND JACOBI TYPE POLYNOMIALS

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Abstract

We study the bispectrality of Laguerre and Jacobi type polynomials, which we define by taking linear combinations of a fixed number of consecutive Laguerre or Jacobi polynomials, respectively. These polynomials are eigenfunctions of higher-order differential operators and include, as particular cases, the Krall-Laguerre and the Krall-Jacobi polynomials. We show that these polynomials always satisfy higher-order recurrence relations (i.e., they are bispectral). We also prove that the Krall-Laguerre and the Krall-Jacobi families are the only Laguerre and Jacobi type polynomials which are orthogonal with respect to a measure in the real line. This is a joint work with Antonio J. Durán.

A NEW APPROACH TO HOMOGENISATION OF PERIODIC QUANTUM GRAPHS

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Abstract

We will discuss the homogenisation problem on periodic quantum graphs embedded into a d-dimensional space. We will show how the results pertaining to high-contrast graphs periodic along a single axis (Cherednichenko, Ershova, Kiselev, SIAM J. App.

Math., 2019) extend to the general periodic context. In so doing, we will further develop the general operator-theoretical approach to high-contrast homogenisation which transparently establishes the connection of high-contrast composites with time-dispersive and spatially dispersive media (and thus with metamaterials). This further allows to suggest a new approach in the study moderate-contrast homogenisation, leading to a concept of a generalised spectral germ and allowing to drop an important restriction imposed by Birman and Suslina.

RICH SPECTRAL PROPERTIES OF PERIODIC QUANTUM GRAPHS

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Abstract

Rich spectral properties of periodic quantum graphs Pavel Exner Doppler Institute for Mathematical Physics and Applied Mathematics, Prague Spectra of periodic quantum systems are usually expected to be absolutely continuous, consisting of bands and gaps, the number of the latter being determined by the dimensionality. If the configuration space topology is nontrivial, however, other possibilities may arise as it will be illustrated using simple examples. In particular, the spectrum may then have a pure point character, and also that it may have only a finite but nonzero number of open gaps. Furthermore, motivated by recent attempts to model the anomalous Hall effect, vertex couplings that violate the time reversal invariance will discussed and it will be shown that their spectral properties are influenced by the vertex degree parity. Finally, we will consider the situation when some graphs links shrink to a point and show that even if the spectra converge as sets, the convergence may be rather non uniform.

GAUGE INVARIANT HELICITY CONTINUITY EQUATION

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Abstract

The derivation of the helicity continuity equation in electromagnetic theory is performed without specifying a gauge. In contrast with previous proposals, the form of this equation is shown to be gauge invariant without invoking a Helmholtz decomposition. The helicity and its flow, the latter associated with the spin in quantized fields, involve two sets of a vector and a scalar potential, where each set can independently undergo a gauge transformation. There are alternative definitions of the helicity and flow densities that arise from different grouping of terms in the continuity differential equation. The various definitions acquire an unambiguous meaning, depending on the gauge and the physical context. The helicity density, defined as

$$\varrho^{(2)}_{\mathbf{AC}} := \sqrt{\frac{\varepsilon}{\mu}}\,(\mathbf{A}\cdot\mathbf{B} - \mathbf{C}\cdot\mathbf{E})$$

and flow density

$$\boldsymbol{J}^{(2)}_{\mathbf{AC}} := \sqrt{\frac{\varepsilon}{\mu}}\,(\mathbf{E} - \nabla\phi_A)\times\mathbf{A} + \frac{1}{\mu\sqrt{\mu\varepsilon}}\,(\mathbf{B} - \nabla\phi_C)\times\mathbf{C}\,,$$

include all the rotational content of the free fields regardless of the gauge. In free space, these quantities satisfy a gauge invariant conservation equation without gauge-fixing source terms. A further asset of the present formulation is that charge and current source terms can be readily incorporated. The helicity source terms are of the form

$$\frac{1}{\sqrt{\mu\varepsilon}}\left(\mathbf{B}\cdot\int\mathbf{J}dt - \int\mathbf{B}dt\cdot\mathbf{J}\right)$$

An helicity continuity equation in terms solely of transverse fields is derived in the Coulomb gauge.

RESULTS ON THE ENTANGLEMENT ENTROPY IN THE XXZ SPIN MODEL

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Abstract

In this talk, we will discuss recent progress on the entanglement entropy for the XXZ spin model. For the finite chain Λ, Beaud and Warzel showed that the entanglement entropy of a generic state ψ from the low-energy ("droplet") regime satisfies a logarithmically corrected area law $S(\psi) \leq C \log |B|$ with respect to a bipartition $\Lambda = B \cup B^c$. In a joint work with H. Abdul-Rahman (U of Arizona) and G. Stolz (U of Alabama at Birmingham), we showed how to extend this result to higher–energy states. In another joint project with R. Schulte (LMU Munich), we are currently trying to show that these bounds are optimal. To this end, we study low-energy states of the free XXZ Hamiltonian on the ring.

RESEARCH ON NORMAL STRUCTURE IN A BANACH SPACE VIA SOME PARAMETERS IN ITS DUAL SPACE

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Abstract

Let X be a Banach space and X^* be its dual. In this talk, we show relationships among some parameters in X^*: ϵ-nonsquareness parameter, $J(\epsilon, X^*)$; ϵ-boundry parameter, $Q(\epsilon, X^*)$; the modulus of smoothness, $\rho_{X^*}(\epsilon)$; and ϵ-Pythagorean parameter, $E(\epsilon, X^*)$),

and weak orthogonality parameter, $\omega(X)$ in X that imply uniform norm structure in X. Some existing results in Fixed Points Theory are extended.

SPECTRAL TRANSFORMATIONS OF MATRIX ORTHOGONAL POLYNOMIALS: ALGEBRAIC PROPERTIES

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Abstract

In this contribution, we present recent results related to spectral transformations of matrix orthogonal polynomials. In particular, we will focus our attention in the so-called Uvarov and Christoffel transformations of orthogonality matrix measures supported on the unit circle, and study algebraic properties such as connection formulas and factorizations of the associated CMV and Hessenberg block matrices, that represent the multiplication operator with respect to the corresponding orthogonal basis.

LEVINSON THEOREM FOR A DISCRETE SYSTEM

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Abstract

A discrete matrix Schrödinger equation with short-rank potential on the line is considered, explicit formulas for the Scattering Matrix are derived and an analysis of the Scattering Matrix is developed in order to prove the continuity of the Matrix at the thresholds, this analysis allows us to derive a formula that counts the eigenvalues of the associated operator in terms to the change in the argument of the determinant of the scattering matrix, that

formula is known as Levinson Theorem.

EXISTENCE OF QUEER POISSON BRACKETS ON BANACH SPACES

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Abstract

There are many pitfalls in the differential geometry of infinite dimensional manifolds. Even in the most regular Banach (or even Hilbert) case many results from finite dimensions do not hold. For example not every symplectic form defines an isomorphism between tangent and cotangent bundle and it leads to difficulties in definition of Poisson structures. Another problem comes from the fact that twice dual of a Banach space is not in general canonically isomorphic to the original space. In this talk I will discuss two other kinds of problems – one originating from the lack of bump functions and the other from inequivalence of various method of defining a tangent vector. It will lead to the existence of Poisson brackets which do not come from Poisson tensors.

The talk is based on the paper: Queer Poisson brackets, J. Geom. Phys. 132 (2018), 358-362.DOI:10.1016/j.geomphys.2018.06.013 (with D. Beltita, A.-B. Tumpach).

REDUCIBILITY OF THE SCHRÖDINGER EQUATION ON A ZOLL MANIFOLD WITH UNBOUNDED POTENTIAL

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Abstract

In this talk I will present some recent results on reducibility for linear Schrödinger equations with quasi-periodic in time potential in different physical context: on the d-dimensional torus, on the whole euclidian space with a confining potential and on a Zoll manifold. Reducibility consists in proving that the original non-autonomous system (i.e. with a time dependent Hamiltonian) can be conjugated to an autonomous one. I will focus mainly on Zoll manifold and I will explain why we can manage unbounded perturbations, namely quasi-periodic in time pseudo-differential perturbation of order less or equal than 1/2.

SPECTRAL GAPS IN GRAPHENE STRUCTURE

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Abstract

I will report on recent results on the spectral analysis of a model of graphene in magnetic fields with constant flux through every hexagonal comb. In particular, we provide a rigorous foundation for self-similarity by showing that for irrational flux, the spectrum of graphene is a zero measure Cantor set. I will also talk about Hausdorff dimension of the spectrum, spectral decomposition and Dirac points. This talk is based on joint works with S. Becker, J. Fillman and S. Jitomirskaya.

WAVES IN PLASMAS: SOME MODELS AND A SINGULARITY

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Abstract

In this talk, I will first discuss some models for wave propagation in a plasma. I will then discuss two methods to the study a singularity at a so-called upper hybrid resonance. The first method was introduced in a joint project with Ricardo Weder, and is based on an integral formulation, while the second one relies on a differential formulation.

RESONANCES IN THE PAULI-FIERZ MODEL

Diego Alejandro Iniesta Miranda

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Abstract

In recent years, spectral properties of nonrelativistic atoms minimally coupled to the quantized electromagnetic field (studied through the Pauli-Fierz Model) have been investigated. Results such as the existence of ground states and resonances in the case of singular coupling have been proved using the spectral renormalization group technique. In this talk, we present an alternative proof of the existence of resonances for the Pauli-Fierz model applying multi-scale analysis.

QUANTUM FIELD THEORY WITH DYNAMICAL BOUNDARY CONDITIONS AND THE CASIMIR EFFECT

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Abstract

We discuss the quantization of a Klein-Gordon field in a region with boundary with dynamical boundary conditions, which are

prescribed by a boundary field equation. We then appropriately define the local energy of the field, which allows us to study the Casimir effect in this model.

STOCHASTIC SCHRÖDINGER EQUATION WITH DIRICHLET BROWN-NOISE BOUNDARY CONDITIONS

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Abstract

We consider the stochastic nonlinear Schrödinger equations on a half-line with Dirichlet brown-noise boundary conditions. We establish the global existence and uniqueness of solutions to the initial-boundary value problem with values in H^1. We are also interested in the regularity behavior of solutions near the origin, where the boundary data are highly irregular.

MELLIN PSEUDODIFFERENTIAL OPERATORS WITH NON-REGULAR SYMBOLS AND THEIR APPLICATIONS

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Abstract

The talk is devoted to applications of Mellin pseudodifferential operators with non-regular symbols to studying singular integral operators with piecewise quasicontinuous coefficients on weighted Lebesgue spaces with Muckenhoupt weights. A localization of a class of Muckenhoupt weights to power weights, which is related

to the Allan-Douglas local principle, is elaborated by using quasicontinuous functions and Mellin pseudo-differential operators with non-regular symbols. A Fredholm symbol calculus for Banach algebras of singular integral operators with piecewise quasicontinuous coefficients on weighted Lebesgue spaces with a subclass of Muckenhoupt weights is con- structed and a Fredholm criterion for the operators in these algebras is obtained. Applications to convolution type operators with non-regular symbols are also considered.

ON THE INSTABILITY OF THE ESSENTIAL SPECTRUM FOR BLOCK JACOBI MATRICES

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Abstract

We are interested in the phenomenon of the essential spectrum instability for a class of unbounded (block) Jacobi matrices. We give a series of sufficient conditions for the matrices from certain classes to have a discrete spectrum on a half-axis of a real line. An extensive list of examples showing the sharpness of obtained results is provided.

This is a joint work with S. Naboko.

INVERSE PROBLEMS FOR QUANTUM GRAPHS

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Abstract

To solve the inverse spectral problem for the Schrödinger equation on a metric graph one needs to determine:

- the metric graph;
- the potential in the Schrödinger equation;
- the vertex conditions (connecting the edges together).

The inverse problem is solved completely in the case of trees under mild restrictions on the vertex conditions. The main tool is a combination of the boundary control and M-function approaches to inverse problems. These two approaches are essentially equivalent in the case of single interval, but their different features may be effectively exploited to solve different partial inverse problems for trees. The **bunch cutting procedure** allows one to reduce the tree step-by-step by removing edges and vertices close to the boundary.

To solve the inverse problem for graphs with cycles we propose to use **magnetic boundary control** and **magnetic M-functions** where spectral data for a fixed potential are considered as functions of the magnetic fluxes through graph cycles. To solve the inverse problem we use **cycle opening procedure** mapping spectral data for arbitrary graphs with cycles to spectral data for trees on the same edge set. The graph and potential are reconstructed assuming so far standard vertex conditions.

THE ESSENTIAL NUMERICAL RANGE OF OPERATORS AND PENCILS

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Abstract

We discuss several alternative characterisations of the essential numerical range for an unbounded operator on a domain in a Hilbert space. Unlike the bounded case, studied by Stampfli and Williams in the 1960s, these definitions are no longer equivalent. We examine how they differ and which one(s) are of greatest use for applications. One of these definitions turns out to be the natural non-selfadjoint replacement for the extended essential spectrum used by Davies, and by Levitin and Shargorodsky, to

analyse spectral pollution when approximating self-adjoint operators using projection methods. We also examine generalisations to linear operator pencils and generalised Morawetz tricks. These allow us to establish, e.g., substantial improvements of a result of Lewin and Sere on spectral pollution for Dirac systems.

Nota: This is joint work with Sabine Boegli (Durham) and Christiane Tretter (Bern).

TIME-DEPENDENT APPROACH TO THE DIFFRACTION PROBLEM BY WEDGES

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Abstract

We consider time-dimensional diffraction of the plane wave e^{ik_0x} by ideal wedges Q of magnitude $\phi \in [0, \pi)$ with DD, NN and DN boundary conditions. In the case when $\phi = 0$ this problem is the famous Sommerfeld diffraction problem by a half-plane. We call this type of diffraction *stationary diffraction* in contrast to *nonstationary diffraction*.

Stationary diffraction problems are described, generally speaking, by ill-posed BVP for the Helmholtz equation in the complement of Q. The ill-posedness consists, in particular, in the fact that these problems admit an infinity of solution and in order to choose the "right" solution, physical ideas are needed. There are many ways to make these problems well-posed, for example using the operator methods [1]. Our approach is based on the representation of stationary diffraction as the limit of the nonstationary one. In this case the incident plane wave is a nonstationary one depending on time t: $e^{-i\omega_0 t - k_0 x} F(\omega_0 t - k_0 x)$, where $F(s)$ is the profile function which determines the form of such incident wave and $F(s) \to 1$, as $t \to \infty$. The last condition means that the amplitude of the incident nonstationary wave tends to the stationary incident wave. The goal of this approach is to prove that

for solutions of the nonstationary diffraction problems the following also holds: their amplitudes tend to the solutions of the corresponding stationary problems. This is the Limiting Amplitude Principle, and it gives another way to choose the "right" solution. It turns out that the nonstationary initial boundary problem is well-posed in some functional class. This is partly due to the fact that the nonstationary problem is reduced (after the Fourier-Laplace transform in t) to the stationary one with strictly elliptic Helmholtz equation, whose wave number is complex.

To prove the well-posedness for the nonstatinary diffraction problem and to solve it in a explicit form, we use the Method of Automorphic functions on Complex Characteristics [2].

[1] Castro LP, Kapanadze D. Wave diffraction by wedges having arbitrary aperture angle. J Math Anal Appl. 2015;421(2):1295-1314.

[2] A. Komech, A. Merzon, (2019) Stationary diffraction by wedges. In: Lecture Notes in Mathematics, vol 2249. Springer, Cham.

Maxwell's equations and the $(D + M^{alpha})$ operator

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Abstract

In [1] it was shown that, when the vectors of the electromagnetic field do not depend on time the Maxwell system is equivalent to Moisil-Teodorescu system

$$(D + M^{\vec{\epsilon}})\vec{\mathcal{E}} = -\frac{\rho}{\sqrt{\epsilon}}$$

$$(D + M^{\vec{\mu}})\vec{\mathcal{H}} = -\frac{\sqrt{\mu}}{\boldsymbol{j}}$$

Using the ideas presented in [2], one can construct operators $\boldsymbol{T}$ and $\widetilde{\boldsymbol{T}}$ such that the Moisil-Teodorescu system can be written as

$$(D + M^{\alpha})\boldsymbol{T} = \widetilde{\boldsymbol{T}}D$$

whenever ϵ and μ are functions admitting separation of variables. A similar transformation is also valid for free force magnetic fields.
Joint work with M. Porter and V. Kravchenko

[1] V.V Kravchenko Applied Quaternionic Analisis. Heldermann Verlag, 2003.
[2] H. Campos, V.V Kravchenko and S. Torba Transmutations L-bases and complete families of solutions of the Stationary Schödinger equation in the plane. Journal of Mathematics Analysis and Applications, 389, 2012.

INFINITE QUANTUM GRAPHS

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Abstract

We consider the Laplacian operator with continuity and Kirchhoff conditions on infinite metric graphs, whose restriction to the space of all compactly supported L^2-functions is symmetric and positive semi-definite, but need not be essentially self-adjoint. We present conditions of geometric-combinatorial flavor that guarantee the uniqueness of self-adjoint extensions. Under a thinness assumption of the graph at infinity, we can fully characterize those extensions that generate a Markovian semigroup; conversely, we can describe those graphs for which such an extension is unique.

THE TRANSMISSION TIME AND LOCAL INTEGRALS OF MOTION FOR DISORDERED SPIN CHAINS

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Abstract

We investigate the relationship between the zero-velocity Lieb-Robinson bounds and the existence of local integrals of motion (LIOMs) for disordered quantum spin chains. We also study the effect of dilute random perturbations on the dynamics of many-body localized spin chains. Using a notion of transmission time for propagation in quantum lattice systems we demonstrate slow propagation by proving a lower bound for the transmission time. This result can be interpreted as a robustness property of slow transport in one dimension. (Joint work with Jake Reschke)

LONG-RANGE SCATTERING FOR THE NLS WITH EQUIATION POTENTIAL

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Abstract

In this talk, we consider the cubic nonlinear Schrödinger equation with an external potential. We prove the existence of modified scattering for this model, that is, linear scattering modulated by a phase. Our approach is based on the spectral theorem for the perturbed linear Schrödinger operator and a factorization technique, which allows us to control the resonant nonlinear term. This approach requires a detailed and subtle study of the low-energy properties of the scattering data. Therefore, we study the cases of generic and exceptional potentials separately. The exceptional case is more delicate: we need some parity assumptions in order to control the small-energy behavior of the scattering coefficients and of the wave functions.

HERGLOTZ FUNCTIONS, THE FOURIER EXTENSION OPERATOR AND CHARACTERIZATIONS OF FUNCTIONS IN THE SOBOLEV SPACES ON THE SPHERE

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Abstract

The classical Fourier extension operator $W(f)$ in the sphere is well defined for any distribution in the sphere and it is a solution of the Helmholtz equation in the entire Euclidean space. A natural and very used situation is when f belongs to L^2 of the sphere, then $W(f)$ is called a Herglotz wave function. There are several characterizations of solutions of the Herglotz wave functions, starting with the classical one proved by Hartman and Wilcox in the 60's. In this talk I will speak about the characterizations of solutions of the Helmholtz arising from the Fourier extension operator spaces of distributions in the unit sphere and in particular from elements in the Sobolev spaces in the sphere. The results presented in this talk require non-differential characterizations of the Sobolev spaces in the sphere. This is done using multivariable square functions in the flavor of the Littlewood-Paley theory.

ORBITAL STABILITY OF STANDING WAVES FOR A NONLINEAR SCHRÖDINGER EQUATION WITH ATTRACTIVE DELTA POTENTIAL AND DOUBLE POWER REPULSIVE NONLINEARITY

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Abstract

A nonlinear Schrödinger equation with an attractive (focusing) delta potential and a repulsive (defocusing) double power nonlinearity in one spatial dimension is considered. It is shown, via explicit construction, that both standing wave and equilibrium solutions do exist for certain parameter regimes. In addition, it is proved that both types of wave solutions are orbitally stable under the flow of the equation by minimizing the charge/energy functional. This is a collaboration with C. Hernández Melo (Maringá) and J. Angulo Pava (Sao Paulo).

INVERSE SPECTRAL PROBLEMS VIA TRUNCATED TOEPLITZ OPERATORS

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Abstract

Recently found methods based on the use of truncated Toeplitz operators produce new examples and algorithms in the area of inverse spectral problems for differential operators. In my talk I will discuss applications of these methods to canonical Hamiltonian systems.
The talk is based on joint work with Nikolai Makarov

FREDNOLM THEORY OF DIFFERENTIAL OPERATORS ON INFINITE GRAPHS

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Abstract

Let Γ be an infinite metric oriented graph embedded in $\mathbb{R}^n$, and $\mathcal{E}$, $\mathcal{V}$ be infinite countable sets of edges $e \in \mathcal{E}$ and vertices $v \in \mathcal{V}$ of Γ. The graph is equipped with a differential operator

$$Au(x) = \sum_{j=0}^{r} a_j(x) u^{(j)}(x)\,, \quad x \in \Gamma \setminus \mathcal{V}.$$

with piece-wise smooth coefficients a_j, $j = 0, \dots, r$, and general connection operators at the vertices $v \in \mathcal{V}$

$$B_k u(v) = \sum_{j=0}^{m_k} b_{jk}(v) u^{(j)}_{\mathcal{E}_v}(v) \ \in \mathcal{C}^{d(v)}\,.$$

with $v \in \mathcal{V}$, $k = 1, \dots, m$, where $d(v)$ is the number of edges incident to v; b_{jk} are $d(v) \times d(v)$ complex matrices,

$$u^{(j)}_{\mathcal{E}_v}(v) = (u^{(j)}_1(v), \dots, u^{(j)}_{d(v)}(v)) \in \mathbb{C}^{d(v)}.$$

where $u^{(j)}_k$ are the limit values at the vertex $v \in \mathcal{V}$ of the derivatives of $u^{(j)}_k$ taken along the edges $e_k \in \mathcal{E}_\mathcal{V}$ according to their orientation.We associate with the operator A and the operators B_k, $k = 1, \dots, m$ an operator $\mathfrak{A} = (A, B_1, B_2, \dots, B_m)$ acting from the Sobolev space $H^s(\Gamma)$ to the space $H^{s-r}(\Gamma) \oplus \mathcal{L}^2(\mathcal{V})^m$, where $\mathcal{L}^2(\mathcal{V}) = \oplus_{v \in \mathcal{V}} \mathbb{C}^{d(v)}$. We study the smoothness, and exponential behavior at infinity of solutions of the equation $\mathfrak{A}u = (f, \phi_1, \dots, \phi_m)$ and for periodic graphs we obtain the necessary and suffcient conditions of the Fredholmness of $\mathfrak{A}$, and a description of the essential spectrum of the realization of $\mathfrak{A}$ in $L^2(\Gamma)$. We give applications of these results to the Schrödinger operators on periodic graphs with general conditions at the vertices.

[1] V.Rabinovich, Fredholm theory of differential operators on periodic graphs, Mathematiclal Problems in Quantum Physics, October 8-11, 2016, Atlanta, Georgia, Contemp. Math., 717, AMS, 2018

[2] V.Rabinovich, On the essential spectrum of quantum graphs, Integr. Equ. Oper. Theory 88, 339–362, 2017

ANALYTICITY PROPERTIES OF THE SCATTERING MATRIX FOR QUASI-ONE-DIMENSIONAL DISCRETE SCHRÖDINGER OPERATORS

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Abstract

Explicit formulas for the scattering matrix and the time delay of a quasi-one-dimensional discrete Schroedinger operators with a potential of finite support are derived. By a careful analysis of the band edge singularities, it is shown that the scattering matrix has an analytic extension at the band edges. This allows to prove a Levinson-type theorem. The main algebraic tool is a new plane wave representation of the transfer matrices.
Based on a joint work with M. Ballesteros and G. Franco.

A FORMULA FOR THE SPECTRAL DENSITY FOR A CLASS OF JACOBI MATRICES IN THE CRITICAL CASE

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Abstract

For a class of Jacobi matrices with unbounded entries in the so called critical (double root, Jordan box) case we establish a formula for the spectral density of the matrix in terms of the asymptotics of orthogonal polynomials associated with it. The talk is based on a joint work with Sergey Naboko.For a class of Jacobi matrices with unbounded entries in the so called critical (double root, Jordan box) case we establish a formula for the spectral density of the matrix in terms of the asymptotics of orthogonal polynomials associated with it.
The talk is based on a joint work with Sergey Naboko.

SPECTRAL PORTRAITS IN THE SEMI-CLASSICAL APPROXIMATION OF THE STURM-LIOUVILLE PROBLEM WITH A COMPLEX POTENTIAL

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Abstract

We study the problem on the limit behavior of the discrete spectrum of the Sturm–Liouville problem

$$-\varepsilon y''(x) + P(x,\lambda)y(x) = 0, \qquad y(a) = y(b) = 0,$$

provided that the physical parameter ε tends to zero. It is assumed that λ is the spectral parameter, the function P is polynomial on x with analytic coefficients on λ varying on a domain G in the complex λ-plane $\mathbb{C}$. The case $P(x,\lambda) = p(x) - \lambda$ with complex valued polynomial p corresponds to the usual linear spectral problem. Boundary conditions are formed by arbitrary complex numbers a, b or $\pm\infty$. The cases of finite segment $[a,b]$, semi-axis and the whole line are considered separately. We shall show that in all cases the eigenvalues are concentrated along the so–called limit spectral graph as $\varepsilon \to 0$. We define three type of curves, forming this graph and find the asymptotic formulae for the eigenvalue distribution along the curves of various types. For the case $P(x,\lambda) = p(x) - \lambda$ with real polynomial $p(x)$ and $(a,b) = \mathbb{R}$ these formulae coincide with the well known Bohr-Sommerfeld quantization formulae. In all other cases the formulae are new. Non-self-adjoint Sturm-Liouville problems are often found in mathematical physics. We demonstrate this considering the well-known in hydromechanics Orr-Sommerfeld spectral problem.
The talk is based on the joint works with S.N.Tumanov.

ESTIMATES AND ASYMPTOTIC BEHAVIOR OF THE DISCRETE SPECTRUM OF A DISCRETE PERIODIC SCHRODINGER OPERATOR PERTURBED BY A DECREASING POTENTIAL

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Abstract

in collaboration with E. L. Korotyaev

Consider a discrete periodic Schrodinger operator $H := \Delta + Q$ on a locally finite connected $\mathbb{Z}^d$-periodic graph Γ, embedded in $\mathbb{R}^d$; here Δ is the discrete Laplace operator on Γ, Q — real bounded $\mathbb{Z}^d$-periodic potential on Γ. The operator H is perturbed by a sign-definite decreasing potential V defined on the graph Γ. The potential V has a power-like asymptotics at infinity

$$0 \le V(x) \sim \vartheta(\frac{x}{|x|})|x|^{-d/p}, \quad |x| \to \infty, \quad p > 0. \tag{4}$$

We are interested in the spectrum of operators $H_\pm(t) := H \pm tV$, $t > 0$, arising in spectral gaps of the operator H. Suppose the spectrum of the operator H contains the gap (Λ_+, Λ_-) (possibly semi-infinite). Since the potential V decreases on infinity, the spectrum of operators $H_\pm(t)$, $t > 0$, is discrete in the gap (Λ_+, Λ_-). Eigenvalues of the operators $H_\pm(t)$ move monotonously with increasing t. The main objects of study are counting functions $N_\pm(\lambda, \tau, V)$, $\lambda \in [\Lambda_+, \Lambda_-]$, $\tau > 0$, equaled to the number of eigenvalues of the operators $H_\pm(t)$ passing through the pointλ with increasing t from 0 to τ.

The main result of the work is as follows: *if the perturbation satisfies the condition* (4)*, then the counting functions have power-like asymptotics with respect to large coupling constant for all* $\lambda \in (\Lambda_+, \Lambda_-)$

$$N_\pm(\lambda, \tau, V) \sim \Gamma_p^\pm(\lambda, H, V)\tau^p, \quad \tau \to +\infty. \tag{5}$$

The coefficients $\Gamma_p^{\pm}(\lambda, H, V)$ *can be calculated in terms of the operator* H *and the perturbation* V*. Under the certain conditions the asymptotics* (5) *holds at the edges of the gap.*

TWO NEW TRACE FORMULAE FOR HERMITIAN MATRICES

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Abstract

Trace formulae are important tools in spectral geometry since they connect between spectral information and geometric or dynamical information pertaining to the operator under study. Typically the spectral information is expressed in terms of the spectral density or the spectral counting function. The geometrical information resides in the properties of periodic structures or orbits such as closed walks on graphs, periodic geodesics, etc. Here, two distinct and new trace formula will be presented and discussed, where the "geometric information" stored in the Hermitian matrices are the traces of powers of the matrix, considered as sums over periodic orbits in the matrix entries. Some applications and relations to classical results will be reviewed.

Joint work with Sven Gnutzmann)

TOEPLITZ QUANTIZATION OF ALGEBRAS AND ITS APPLICATION TO THE QUANTUM GROUP $SU_q(2)$

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Abstract

First, I will outline a way to define Toeplitz operators and their corresponding Toeplitz quantization where the symbols come from a not necessarily commutative algebra. Then this will be applied to the special case where the symbols are elements of the quantum group $SU_q(2)$.

Homogenization of hyperbolic equations with periodic coefficients

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Abstract

We give a survey of the results on operator error estimates in homogenization of hyperbolic equations with periodic rapidly oscillating coefficients (Birman and Suslina, 2008; Meshkova, 2017; Dorodnyi and Suslina, 2018, 2019).

In $L_2(\mathbb{R}^d;\mathbb{C}^n)$, we consider a selfadjoint matrix strongly elliptic operator A_ε, $\varepsilon > 0$, given by the differential expression $b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})$. Here $g(\mathbf{x})$ is a periodic bounded and positive definite matrix-valued function, and $b(\mathbf{D})$ is a first order differential operator. We study the behavior of the operators $\cos(tA_\varepsilon^{1/2})$ and $A_\varepsilon^{-1/2}\sin(tA_\varepsilon^{1/2})$, $t \in \mathbb{R}$, for small ε. It is proved that, as $\varepsilon \to 0$, these operators converge to $\cos(tA_0^{1/2})$ and $A_0^{-1/2}\sin(tA_0^{1/2})$, respectively, in the norm of operators acting from the Sobolev space $H^s(\mathbb{R}^d;\mathbb{C}^n)$ (with a suitable s) to $L_2(\mathbb{R}^d;\mathbb{C}^n)$. Here $A_0 = b(\mathbf{D})^* g^0 b(\mathbf{D})$ is the effective operator. We prove sharp-order error estimates and study the question about the sharpness of the results with respect to the norm type and with respect to dependence of estimates on t. The results are applied to study the behavior of the solution $\mathbf{u}_\varepsilon(\mathbf{x},t)$ of the Cauchy problem for hyperbolic equation $\partial_t^2 \mathbf{u}_\varepsilon(\mathbf{x},t) = -(A_\varepsilon \mathbf{u}_\varepsilon)(\mathbf{x},t)$. Applications to the nonstationary equations of acoustics and elasticity are given. The method is based on the scaling transformation, the Floquet-Bloch theory and the analytic perturbation theory.

OVERSAMPLING AND UNDERSAMPLING IN DE BRANGES SPACES

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Abstract

Let $\mathcal{B}$ be a de Branges space of entire functions, whose reproducing kernel is denoted $K_{\mathcal{B}}(z,w)$. Let $S_{\mathcal{B}}$ denote the (maximal) operator of multiplication by the independent variable in $\mathcal{B}$.

A de Branges space $\mathcal{B}$ has the oversampling property if, for every de Branges subspace $\mathcal{A} \subsetneq \mathcal{B}$, there exists a function $J_{\mathcal{AB}} : \mathbb{C} \times \mathbb{C} \to \mathbb{C}$ such that:

(o1) For all $w \in \mathbb{C}$, $J_{\mathcal{AB}}(\cdot, w) \in \mathcal{B}$; moreover, $J_{\mathcal{AB}}(z,w) = J_{\mathcal{AB}}(w,z)^* = J_{\mathcal{AB}}(w^*, z^*)$.

(o2) For some selfadjoint extension $S_{\mathcal{B},\gamma}$ of $S_{\mathcal{B}}$, the identity

$$F(z) = \sum_{\lambda \in \sigma(S_{\mathcal{B},\gamma})} \frac{J_{\mathcal{AB}}(z,\lambda)}{K_{\mathcal{B}}(\lambda,\lambda)} F(\lambda)$$

holds true for every $F \in \mathcal{A}$, where the convergence is absolute and uniform in compact subsets of $\mathbb{C}$.

(o3) The series

$$\sum_{\lambda \in \sigma(S_{\mathcal{B},\gamma})} \frac{|J_{\mathcal{AB}}(z,\lambda)|}{\sqrt{K_{\mathcal{B}}(\lambda,\lambda)}}$$

converges uniformly in compact subsets of $\mathbb{C}$.

On the other hand, $\mathcal{B}$ has the undersampling (or aliasing) property if given any de Branges subspace $\mathcal{A} \subsetneq \mathcal{B}$, there exists a selfadjoint extension $S_{\mathcal{A},\gamma}$ of $S_{\mathcal{A}}$ such that:

(u1) For every $F \in \mathcal{B}$, the series

$$\tilde{F}(z) = \sum_{\lambda \in \sigma(S_{\mathcal{A},\gamma})} \frac{K_{\mathcal{A}}(z,\lambda)}{K_{\mathcal{A}}(\lambda,\lambda)} F(\lambda)$$

converges absolutely and uniformly in compact subsets of $\mathbb{C}$.

(u2) Given a compact subset $\mathbb{K} \subset \mathbb{C}$, there exists $C(\mathbb{K}) > 0$ such that

$$\left|\tilde{F}(z) - F(z)\right| \leq C(\mathbb{K}) \, \|(I - P_{\mathcal{AB}})F\|_{\mathcal{B}}$$

for all $F \in \mathcal{B}$, where $P_{\mathcal{AB}} : \mathcal{B} \to \mathcal{B}$ denotes the orthogonal projector onto $\mathcal{A}$.

The notions of oversampling and aliasing stem from the theory of Paley-Wiener spaces. Since Paley-Wiener spaces are particular instances of de Branges spaces, it is natural to ask whether these properties are present in some other classes of de Branges spaces, maybe in all of them. In this talk, we present some results about oversampling and aliasing in de Branges spaces associated with some classes of Schrödinger operators, thus providing a partial answer to the aforementioned question.

This talk is based on joint works with L. O. Silva and A. Uribe.

SERIES REPRESENTATION OF INTEGRAL KERNELS OF TRANSMUTATION OPERATORS AND APPLICATIONS TO EFFICIENT SOLUTION OF SPECTRAL PROBLEMS

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Abstract

Solutions to a one-dimensional Schrödinger equation $-y''+q(x)y = \omega^2 y$ considered on a segment $[-b, b]$ can be represented as images of the solutions to the simplest equation $-y'' = \omega^2 y$ under the action of a transmutation operator T having the form

$$Tu(x) = u(x) + \int_{-x}^{x} K(x,t)u(t)\,dt, \qquad x \in [-b, b].$$

We consider the expansion of the integral kernel K into a series with respect to the system of orthogonal polynomials, present formulas for the coefficients and show that truncated sums can be

used for efficient numerical solution of initial value and spectral problems.

Similar results are presented for the perturbed Bessel equation $-y'' + \frac{\ell(\ell+1)}{x^2} + q(x)y = \omega^2 y$.
The talk is based on the results obtained with V. V. Kravchenko, Luis J. Navarro and R. Castillo-Perez.

ALGEBRAS GENERATED BY TOEPLITZ OPERATORS WHOSE SYMBOLS ARE INVARIANT UNDER A GROUP ACTION

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Abstract

Let $\mathcal{A}^2_\lambda(\mathbb{B}^n)$, $\lambda \in (-1, \infty)$, be the standard weighted Bergman space on the unit ball $\mathbb{B}^n$ in $\mathbb{C}^n$. And let T^λ_a denote the Toeplitz operator, with symbol $a \in L_\infty$, acting on $\mathcal{A}^2_\lambda(\mathbb{B}^n)$. Given an (Abelian) subgroup G of biholomorphisms of $\mathbb{B}^n$, we are looking for a Bargmann type unitary transform of $\mathcal{A}^2_\lambda(\mathbb{B}^n)$ onto a direct sum (or direct integral) of Hilbert spaces

$$R_G : \mathcal{A}^2_\lambda(\mathbb{B})^n \to \bigoplus_\alpha H_\alpha \left(\int_\alpha^\oplus H_\alpha d\alpha \right)$$

and a set of G-invariant symbols

$$S_G = \{a \in L_\infty(\mathbb{B}) : a = a \circ g \text{ for all } g \in G\}$$

such that, for each $a \in S_G$, the operator $R_G T^\lambda_a R^*_G$ leaves invariant each Hilbert space H_α in the above direct sum (direct integral) decomposition. In that way a problem of the characterization of the algebra generated by Toeplitz operators T^λ_a, with $a \in S_G$, reduces to the description of the algebras generated by the operators $R_G T^\lambda_a R^*_G \| _{H_\alpha}$ in each of the spaces H_α. Furthermore certain quantization effects (caused by α tending to infinity) have to be taken into account. In the talk we present several examples of the above approach characterizing various commutative C^*, commutative Banach, and non commutative

C^*-algebras generated by Toeplitz operators for the case of the two-dimensional unit ball. The talk is based on a joint work with Wolfram Bauer.

NON LINEAR-FOURIER TRANSFORM USING LARGE TRANSMUTATION OPERATORS AND SPPS REPRESENTATIONS

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Abstract

In this talk we focus on the direct non-linear Fourier transform for the Non Linear Schrödinger Equation, which reduces to the study of the Zakharov-Shabat (Z-S) system [1,4,5] of the form

$$\begin{pmatrix} v_1'(x) \\ v_2'(x) \end{pmatrix} = \begin{pmatrix} -i\lambda & q(x) \\ -q^*(x) & i\lambda \end{pmatrix} \begin{pmatrix} v_1(x) \\ v_2(x) \end{pmatrix},$$

where $v_{1,2}(x)$ are unknown complex functions, λ is the spectral parameter, the complex-valued function $q(x)$ is the potential, the simbol $*$ represents the complex conjugation and i is the imaginary unit. Since the Z-S system reduces to Sturm-Liouville equations we show, under a few restrictions for the potential, the spectral parameter power series [2] and the analytic approximation of transmutations operators [3] representations for the solutions of the Z-S system and the corresponding nonlinear Fourier coefficients. Finally we show numerical experiments, properties and numerical advantages of each method.

[1] Mark J Ablowitz and Harvey Segur. Solitons and the inverse scattering transform, volume 4. Siam, 1981.

[2] Vladislav V Kravchenko and R Michael Porter. Spectral parameter power series for Sturm–Liouville problems. Mathematical Methods in the Applied Sciences, 33(4): 459–468, 2010.

[3] Vladislav V Kravchenko and Sergii M Torba. Analytic approximation of transmutation operators and applications to

highly accurate solution of spectral problems. Journal of Computational and Applied Mathematics, 275: 1– 26, 2015.
[4] JK Shaw. Mathematical principles of optical fiber communications, volume 76. Siam, 2004.
[5] Mansoor I Yousefi and Frank R Kschischang. Information transmission using the nonlinear fourier transform, part i: Mathematical tools. IEEE Transac- tions on Information Theory, 60(7): 4312–4328, 2014.

TRANSMUTATION OPERATORS AND COMPLETE SYSTEMS OF SOLUTIONS FOR THE D-DIMENSIONAL RADIAL SCHRÖDINGER EQUATION

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Abstract

The aim of this talk is to present an explicit construction of an orthogonal complete system of solutions for the radial Schrödinger equation. We consider in a star-shaped domain Ω of the Euclidean space $\mathbb{R}^d$ (with $d \geqslant 2$) the equation of the form

$$\boldsymbol{S}u(\mathbf{x}) := (\triangle_d - q\,(\|\mathbf{x}\|))\,u(\mathbf{x}) = 0 \quad \text{for } \mathbf{x} \in \Omega, \tag{6}$$

where q is a C^1 function that only depends of the radial component $r = \|\mathbf{x}\|$.

Using the approach of transmutation operators theory, we consider the fact of any solution of (6) can be written in the form

$$\boldsymbol{T}f(\mathbf{x}) = f(\mathbf{x}) + \int_0^1 \sigma^{d-1} G(r, 1-\sigma^2) f(\sigma^2 \mathbf{x}) d\sigma, \tag{7}$$

where h is an harmonic function in Ω [1]. We show that $\boldsymbol{T}$ is a transmutation operator for the pair $\hat{\boldsymbol{S}} := r^2\boldsymbol{S}$ and $\hat{\boldsymbol{L}} := r^2\triangle_d$ and that it is continuous and invertible.

Employing the properties of $\boldsymbol{T}$, we obtain a complete system of solutions $\mathcal{S}$ for (6), that generalizes the concept of harmonic homogeneous polynomial. The completeness of the system $\mathcal{S}$ in the sense of the uniform convergence on compact subsets of Ω is

established, with the Runge property in term of it. In addition, we develop the explicit construction of the system, and for the case that the domain is an open ball centered at the origin, we obtain an orthogonal basis for the Bergman space of L_2-solutions.

[1] R. P. Gilbert, K. Atkinson, *Integral operator methods for approximating solutions of Dirichlet problems.* ISMN Vol. 15, Birkhäuser, Basel, 1970.

This is a joint work with Vladislav V. Kravchenko

ON LIMITING EIGENVALUE OR RESONANCE DISTRIBUTION THEOREMS IN MATHEMATICAL PHYSICS

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Abstract

We will describe several cases where we can state a semiclassical limiting eigenvalue or resonance distribution theorem for clusters of either eigenvalues or resonances after introducing a perturbation of an initial system whose eigenvalues are both degenerated and isolated. This includes the initial result by A. Weinstein on perturbations of the Laplacian on a Riemannian compact manifold all of whose geodesics are closed. We will mainly concentrate on the system given by perturbations of the Landau problem considering the semiclassical limit in two different ways.

SPECTRAL THEORY FOR SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS WITH DISTRIBUTIONAL COEFFICIENTS

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Abstract

We discuss the spectral theory of the first-order system $Ju'+qu = wf$ of differential equations on the real interval (a, b) when J is a constant, invertible skew-Hermitian matrix and q and w are matrices whose entries are distributions of order zero with q Hermitian and w non-negative. We do not require the definiteness condition often made on the coefficients of the equation. Specifically, we construct associated minimal and maximal relations, and study self-adjoint restrictions of the maximal relation. For these we construct Green's function and prove the existence of a spectral (or generalized Fourier) transformation.

This is joint work with Kevin Campbell, Ahmed Ghatasheh, and Minh Nguyen

ESTIMATES FOR EIGENVALUES IN GAPS OF THE ESSENTIAL SPECTRUM

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Abstract

In this talk I will show how bounds for eigenvalues in gaps of the essential spectrum of a linear operator can be obtained. We will apply these results to a one-dimensional Dirac type operator.

ASYMPTOTIC BEHAVIOR OF ORTHOGONAL POLYNOMIALS WITHOUT THE CARLEMAN CONDITION

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Abstract

Our goal is to find an asymptotic behavior as $n \to \infty$ of orthogonal polynomials $P_n(z)$ defined by the Jacobi recurrence coefficients a_n, b_n. We suppose that the off-diagonal coefficients a_n grow so rapidly that the series $\sum a_n^{-1}$ converges, that is, the Carleman condition is violated. With respect to diagonal coefficients b_n we assume that $-b_n a_n^{-1} \to 2\beta_\infty$ for some $\beta_\infty \neq \pm 1$. The asymptotic formulas obtained for $P_n(z)$ are quite different from the case $\sum a_n^{-1} = \infty$ when the Carleman condition is satisfied. In particular, if $\sum a_n^{-1} < \infty$, then the phase factors in these formulas do not depend on the spectral parameter $z \in \mathbb{C}$. The asymptotic formulas obtained in the cases $|\beta_\infty| < 1$ and $|\beta_\infty| > 1$ are also qualitatively different from each other. These results imply, in particular, that the corresponding Jacobi operator has deficiency indices $(1, 1)$ in the first case, while it is essentially self-adjoint in the second case.

LIEB-THIRRING INEQUALITIES FOR FINITE AND INFINITE GAP OPERATORS

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Abstract

In this talk I will overview classical results as well as recent advances on Lieb-Thirring bounds (i.e. eigenvalue power bounds) for discrete eigenvalues of either Jacobi or Schrödinger operators with periodic or almost-periodic background potentials and self-adjoint and non-self-adjoint L^p class perturbations.

A NEW APPROACH TO PROOF THE RIEMANN HYPOTESIS USING A NEW OPERATOR

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Abstract

In this note we present a new approach to proof the Riemann hypothesis one of the most important open problem in pure mathematics using a new operator derived from unitary operator groups acting on Riemann-Siegal function and it uses partition function for Hamiltonian operator.

POSTERS

DISPERSIVE PROPERTIES OF RESONANT STARTIFIED ELASTIC MEDIA

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Abstract

Motivated by the novel approach proposal in [2] to critical-contrast homogenisation for periodic PDEs. We aplied and develop the tecniques adopted in [2] to the theory of linear elasticity for continuous media in the vector case. Specifically, to isotropic materials. It is important to note that although [1, 2, 3] concrete examples of application of this new perspective have been presented, which have been made in the scalar case, however modeling and application of the theory in other problems is not trivial because the implementation depends strongly on the microstructure properties of the medium.

[1] Cherednichenko, K., Ershova, Yu., Kiselev, A. V., 2018. Time-dispersive behaviour as a feature of critical contrast media. arXiv: 1803.09372

[2] Cherednichenko, K., Ershova, Yu., Kiselev, A. V., 2018. Effective behaviour of critical- contrast PDEs; Micro-resonances, Frequency conversion, and time-dispersive media. I. In preparation.

[3] Cherednichenko, K., Ershova, Yu., Kiselev, A. V.,and Naboko, S., 2018. Unified ap- proach to critical-constrast homogenisation with explicit links to time-dispersive media. arXiv: 1805.00884.

On the Path Integral for Non-Relativistic Quantum Electrodynamics

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Abstract

Based on previous work done by W. Ichinose, a rigorous model for the path integral in the context of nonrelativistic quantum electrodynamics is studied. We begin with a Lagrangian that corresponds to the classical theory of electromagnetism, which allows us to define a classical action. We restrict the theory to a box with periodic boundary condition in order to expand the electromagnetic potential as Fourier series. The integral is defined as a limit of integrals over picewise constant paths which go over the possible positions of the particle and the possible Fourier coefficients. We prove that this limit is a solution of the Schrödinger equation for the respective quantum Hamiltonian.

ON THE LOCATION OF THE ZEROS OF MATRIX ORTHOGONAL POLYNOMIALS

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Abstract

In this contribution, we review some properties related to the location of the zeros of matrix orthogonal polynomials on the unit circle. In particular, we are interested in the location of the zeros of their derivatives. This can be helpful to determine relative asymptotic results of families of matrix polynomials associated with spectral transformations of matrix measures

DIFFERENTIAL OPERATORS WITH WHITE-NOISE

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Abstract

We study differential operators with random coefficients in terms of the white noise; this includes an operator with a random potential. Using bilinear forms we propose finding explicitly the Green kernel of these operators. To do so we appeal to theory of Sturm-Liouville.

INVERSE SPECTRAL ANALYSIS FOR A CLASS OF BAND SYMMETRIC MATRICES

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Abstract

An inverse spectral problem is solved for a class of band symmetric matrices. We give the necessary and sufficient conditions so that a matrix-valued function to be a spectral function of the corresponding operator. Additionally, we give an algorithm for recovering the matrix from the spectral function. The class studied corresponds to operators arising from difference equations with usual and inner boundary conditions. This work was done in collaboration with Mikhail Kudryavtsev and Luis O. Silva.

Henstock-Kurzweil Sobolev space

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Abstract

Sobolev spaces, defined as integrable functions whose weak derivative is integrable, are useful to ensure the existence and uniqueness of weak solutions to problems in ordinary differential equations. In this talk, we will show some properties of these spaces

if we consider the integrable functions in the sense of Henstock-Kurzweil, which allows generalizing some classic results of the Lebesgue integral.

STURM-LIOUVILLE DIFFERENTIAL EQUATIONS WITH HENSTOCK-KURZWEIL INTEGRABLE FUNCTIONS

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Abstract

We give sufficient conditions for existence and uniqueness of solutions of the Sturm-Liouville equations involving Henstock-Kurzweil integrable functions as coefficients, these results consider highly oscillatory functions not Lebesgue integrable. Examples in the sense variational and in the numerical framework are provided.
This work was done in collaboration with Tomás Pérez Becerra.